Southern Federal University
I. I. Vorovich Institute of Mathematics, Mechanics,
and Computer Science

Proceedings of
Twelfth China-Russia Conference

NUMERICAL ALGEBRA WITH APPLICATIONS

Proceedings of Lecturers and Young Scientists

5-11 July 2026
Krasnoyarsk – Rostov-on-Don, RUSSIA

Rostov-on-Don – Taganrog
Publishing House of the Southern Federal University
2026

UDC 519.61(063)

AMS(MOS) subject classification: 65A06, 65F10

Editors: Galina V. Muratova, Irina N.Shabas

Numerical Algebra With Applications The China-Russia Conference : Proceedings of Lecturers and Young Scientists (5-11 July 2026, Krasnoyarsk – Rostov-on-Don) ; Southern Federal University. – Rostov-on-Don ; Taganrog : Publishing House of the Southern Federal University, 2026. – 48 p.
ISBN 978-5-9275-5280-1

The 12th China-Russia Conference on Numerical Algebra with Applications (CR-CNAA 2026) continues the tradition of its eleven predecessors and provides a forum for researchers, scientists, engineers, and graduate students to present papers on the latest advances in numerical algebra and computational methods, as well as their applications. It is organized by the I.I. Vorovich Institute of Mathematics, Mechanics, and Informatics of Southern Federal University and Siberian Federal University, with the participation of the Institute of Computational Modeling of the Siberian Branch of the Russian Academy of Sciences and the Institute of Computational Mathematics and Scientific and Technical Computing of the Chinese Academy of Sciences. Topics for CRCNAA 2026 include, among others, preconditioning and iterative methods for systems of linear and nonlinear equations, eigenvalue computation for large and sparse linear and nonlinear systems, parallel computing, randomized numerical algebra, mathematical modeling, and the application of numerical methods to real-world problems.

UDC 519.61(063)

ISBN 978-5-9275-5280-1

CONTENTS

Plenary Lectures

REFINED ERROR BOUNDS FOR RELAXED GREEDY RANDOMIZED KACZMARZ METHODS

Zhong-Zhi Bai
Institute of Computational Mathematics,
State Key Laboratory of Mathematical Sciences,
Academy of Mathematics and Systems Science,
Chinese Academy of Sciences,
Beijing 100190, P.R. China
Email: bzz@lsec.cc.ac.cn

Abstract: By rigorously analyzing the asymptotic convergence property of the relaxed greedy randomized Kaczmarz method, we obtain an upper bound on its asymptotic convergence rate that is sharper than the existing one. Also, this upper bound naturally leads to a strengthened convergence guarantee for the greedy randomized Kaczmarz method. Consequently, this result improves and enriches the convergence theory of the greedy randomized Kaczmarz method with or without a relaxation parameter, when these iteration methods are applied to solve large-scale and consistent system of linear equations.

NUMERICAL IMPLEMENTATION OF THE SEMI-INVERSE METHOD FOR LARGE DEFORMATION PROBLEMS OF COSSERAT-TYPE ELASTIC SHELLS

Mikhail I. Karyakin, Leonid M. Zubov

Southern Federal University, Russia

This study addresses two boundary value problems concerning finite deformations of thin elastic shells, solved using computational algorithms based on the semi-inverse method. The mathematical formulation employs the general nonlinear theory of Cosserat-type elastic shells, in which the shell is modeled as a two-dimensional continuum – a material surface endowed with six rigid-body degrees of freedom at each point. Since displacements and rotation angles of elementary surface elements are not assumed to be small, numerical techniques are required to solve the resulting highly nonlinear systems.

The first problem focuses on the numerical modeling of surface coatings applied to a hollow circular cylinder. The three-dimensional material of the cylinder is described by nonlinear elasticity equations for an isotropic incompressible medium. The coatings possess a field of initial stresses generated by a prescribed initial deformation comprising axial tension/compression, torsion, and radial strain. For the composite system "cylindrical pipe + prestressed coatings," a boundary value problem is solved numerically to analyze the combined effects of internal and external hydrostatic pressures, an axial force, and end torques. The interaction between the pipe's core layer and the coatings is governed by specialized boundary conditions on the inner and outer lateral surfaces, leading to a high-dimensional governing system of nonlinear algebraic equations. The semi-inverse method enables the reduction of the original three-dimensional problem to a sequence of lower-dimensional subproblems. The resulting system is solved using computational algebra techniques, specifically Newton-Raphson iteration and parameter continuation algorithms. The numerical results illustrate the influence of coating prestresses on the mechanical response of the composite cylinder. In particular, the torsional deformation of the pipe induced by internal pressure under zero applied torque is investigated numerically.

The second problem demonstrates the application of the semi-inverse method to analyze the deformation of a thin elastic shell shaped as a helicoid. This approach reduces the original two-dimensional nonlinear boundary value problem to a one-dimensional system of ordinary differential equations governing the radial displacement of shell points and the microrotation angle of its material particles. The BVP for the resulting ODE system is solved numerically via finite difference or shooting methods. A computational tension-compression diagram

is constructed for the helicoid functioning as a spring element in the absence of external torque. The results are compared with two independent approaches: comparison with a reduced analytical model and direct three-dimensional finite element modeling of the helicoid as a solid elastic body using computational algebra libraries. The high degree of agreement between the methods confirms the validity of the proposed numerical-analytical approach.

IMPROVING THE ACCURACY OF OPERATIONAL FORECASTING A SOCIO-ECONOMIC OR TECHNICAL DYNAMICAL MODEL DESCRIBED BY A SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS[1]

Vladimir V. Shaydurov, Viktoriya S.Petrakova

Institute of Computational Modelling of Siberian Branch of Russian Academy of Science;
Russia, Krasnoyarsk

In the talk, we study a method for reconstructing parameters of dynamic model described by a system of ordinary differential equations with these parameters in the right-hand side which are controlled by external circumstances or instructions. To adjust the parameters, in our previous paper [1] we construct the system of algebraic equations which resulting from the approximation of the system of differential equations taking into account the known values of the solution vector at two successive moments in time. This approximation has only the first order of accuracy. Then, due to the smooth change of parameters over time, we simultaneously minimize the residual for the algebraic system and the sum of squares of the parameter differences between two successive moments in time. This led to the regularized system of linear algebraic equations with a positive definite matrix and with a unique set of parameters.

Now we use the approximation of the system of differential equations taking into account the known values of the solution vector at three successive moments in time. This new approximation has the second order of accuracy which leads to increasing the accuracy of the entire forecast as a whole. At the same time, the conservation law (characteristic of the previous two-layer difference scheme) is fulfilled for the participants number. Theoretical justification and numerical experiments confirm the increasing the convergence order for the presented three-layer difference scheme.

Bibliography

1. *V. Shaydurov and V. Petrakova* Correction of Dynamic Model Parameters by Observed Data. Lobachevskii Journal of Mathematics, 2025, Vol. 46, No. 2, pp. 879-887.

[1]The study was implemented in the framework of the Basic Research Program at HSE University in 2026 year.

AUTOMATED VORTEX DETECTION USING LIMITED FLOW VELOCITY DATA AND AN ANALYTICAL VORTEX MODELS

Vasily N. Govorukhin
Institute of Mathematics, Mechanics and Computer Science
Southern Federal University
8a Milchakova St., Rostov-on-Don 344090, Russia
vngovoruhin@sfedu.ru

Abstract In this talk, an automated approach to solving the inverse problem of vortex identification in two-dimensional flows of complex structure using a limited set of velocity vectors is proposed and validated. The goal of the method is to estimate the number of vortices and their key parameters: center coordinates, intensities, and radii.

The flow is represented by a set of model vortices with known analytical induced velocities. Vortex parameters are found by minimizing a functional that measures the discrepancy between the original and model velocity fields. The algorithm includes: I) sliding multi-scale scanning with single-vortex identification per window; II) clustering to remove duplicates; III) refinement of clustered vortices by locally re-solving the inverse problem for one vortex. Steps II and III are iterated until the vortex count stabilizes.

Validation on real surface currents from the Black Sea and the southern Pacific Ocean. The algorithm's performance is confirmed by several criteria. First, interpolated velocity at the detected vortex centers was very low (mostly <0.01), accurately identifying near-zero-velocity points. Second, for a circular contour of radius 0.1 around each vortex, circulation computed from the original data and from the Rankine model showed a relative difference below $47\%(mostly < 20\%)$, with matching rotation directions. Third, halving the input data still preserved 28 out of 32 Black Sea vortices; the lost ones were weak or near the domain boundary, and the centers of the remaining vortices stayed close to the full-data results. Finally, applied to consecutive six-hour snapshots, the algorithm successfully captured the evolution of the vortex field – including changes in vortex count and positions over time.

Low computational cost makes the method suitable for large datasets, from laboratory experiments to engineering applications and ocean studies. It can also provide initial guesses for other identification methods and starting data for flow forecasting.

CURRENT COMPUTATIONAL ALGORITHMS AND THEIR ACCELERATION METHODS[1]

Galina V. Muratova, Tatiana S. Martynova
Vorovich Institute of Mathematics, Mechanics and Computer Science
Southern Federal University
8a Milchakova St., Rostov-on-Don 344090, Russia
muratova@sfedu.ru, martynova@sfedu.ru

Abstract

The results of research aimed at developing new computational algorithms and methods for accelerating them are presented for a wide range of applied problems. The focus is on mathematical modeling, numerical methods, and coding theory.

The approaches that go beyond traditional parallelization are investigated. These include quantization, which involves switching from calculations with long numbers to arithmetic with short integer or real data types, which significantly improves performance.

One of the key areas of application is solving high-order systems of linear algebraic equations (SLAEs) (up to a million or more unknowns) arising in finite element approximation of mathematical physics problems. Multigrid methods (both geometric and algebraic – AMG) are selected as the most effective approach. They have an $O(N)O(N)$ complexity, convergence rate independence from the problem size, and invariance to the reordering of the unknowns.

The generalized eigenvalue problem $Az = jMzAz = jMz$ with symmetric positive-definite matrices is considered. The conjugate gradient method (PCG) with a preconditioner based on the smoothed aggregation algebraic multigrid method (SA-AMG) is used.

The results of numerical experiments are presented.

[1]The research was funded by a State assignment project FENW-2026-0031 at Southern Federal University

DIGITAL TWINS OF HEAT AND MASS TRANSFER IN PERMAFROST REGIONS: BEARING CAPACITY OF SOILS AND DEVELOPMENT STRATEGY OF OIL AND GAS FIELDS[1]

Mikhail Yu. Filimonov

Krasovskii Institute of Mathematics and Mechanics, Ekaterinburg, Russia, 16 S.Kovalevskaya Str., Yekaterinburg, 620077 Russia

fmy@imm.uran.ru

In permafrost regions, climatic and technogenic impacts lead to permafrost degradation, requiring effective thermal stabilization methods for the design and operation of engineering structures. For capital facilities on pile foundations, the key challenge is to maintain the frozen state of the soil around the piles to ensure the bearing capacity of the foundations. In the oil and gas industry, an additional task arises: optimizing the spacing between production and injection wells at pad sites, taking into account their thermal interaction. Both classes of problems are described by heat and mass transfer models in the "foundation/well - soil - atmosphere" system. This paper proposes the use of digital twins that allow:

1. predicting changes in the thermal regime and bearing capacity of soils under engineering structures (including assessment of the efficiency of seasonal cooling devices (SCDs) and ventilated basements)

2. developing an operation strategy for oil and gas fields based on numerical modeling, in particular determining optimal well spacing and operating regimes

The modeling results can be integrated into regulatory temperature monitoring protocols and into adaptive infrastructure management schemes in permafrost regions.

The study was conducted in collaboration with N.A. Vaganova, I.M. Filimonova, D. Zh. Shamugia.

[1]The research was funded by a grant of the Russian Science Foundation N 25-21-00776.

ON ALGEBRAIC-TOPOLOGICAL MULTIGRID DOMAIN DECOMPOSITION METHODS (ADDM-MG)[1]

Valeriy P. Ilyin

Institute of Computational Mathematics and Mathematical Geophysics,

Siberian Branch of the Russian Academy of Sciences

This paper examines preconditioned iterative methods in Krylov subspaces for solving large systems of linear algebraic equations (SLAEs) with sparse matrices obtained by approximating multidimensional initial value or initial-boundary value problems on structured or unstructured grids. Domain decomposition and the formation of vector-matrix data structures are accomplished by partitioning the nodes of the original grid into four subsets based on their membership in different topological primitives of the macrogrid: macronodes, macroedges, macrofaces, and macrocells (subdomains). With the appropriate ordering of the algebraic objects, the SLAE matrix takes a block-tridiagonal form, and its preconditioning is performed using incomplete factorization and banded approximation of inverse matrices, based on the principle of generalized compensation. To solve algebraic systems, single- or two-level iterative processes are constructed in Krylov subspaces. Symmetric systems are implemented using deflationary conjugate direction methods with accelerated convergence by expanding the set of basis vectors. In non-stationary problems, numerical solutions from previous time steps are used. Preconditioning of algebraic subsystems in subdomains is performed using multigrid algorithms, defined as a recursive application of two-grid algorithms, which are based on a topological partitioning of the node set into four subsets, similar to a macrogrid but at the microlevel. As a result, a unified ADDM-MG preconditioner with a nested block tridiagonal structure is formed for the original SLAE. Parallelization and high-performance algorithms are discussed, with preliminary experimental results illustrated. The development of new iterative processes for the efficient solution of a wide range of mathematical modeling problems is considered.

[1]The research was conducted within the framework of the state assignment of the Institute of Mathematical and Mathematical Geophysics of the Siberian Branch of the Russian Academy of Sciences, topic FWNM-2025-0001.

HOMOGENIZATION OF PIEZOELECTRIC METAMATERIALS WITH PERIODIC BOUNDARY CONDITIONS: MATHEMATICAL ISSUES, FINITE ELEMENT IMPLEMENTATION AND NUMERICAL RESULTS[1]

Andrey Nasedkin

Institute of Mathematics, Mechanics and Computer Science, Southern Federal University, Rostov on Don, 344090, Russia
E-mail: avnasedkin@sfedu.ru

Abstract

In recent years, interest in the design of highly porous lattices composed of individual cellsвЂ”metamaterialsвЂ”has grown significantly. Such structures can be created using modern additive manufacturing or 3D printing from a variety of materials, both conventional and those with complicated properties, in particular piezoelectric. Various polarization methods have also been proposed for piezoelectric lattices and metamaterials, enabling the production of both uniformly polarized lattices and lattices with controlled inhomogeneous polarization properties. In the latter cases, it is possible to create structures with rod or shell blocks in which the material moduli have different values and varying anisotropy, both in their local coordinate systems and in a common global coordinate system.

This paper discusses the mathematical aspects of setting and solving simplified polarization problems and homogenization problems for piezoelectric metamaterials with periodic boundary conditions. It provides energetic justification, proves the symmetry properties, positive definiteness of the effective material moduli matrices, and the equality of the effective piezoelectric moduli of the direct and inverse piezoelectric effects. The finite element method is used to numerically solve the homogenization problems, demonstrating that all the main stages of the method can be implemented with finite element matrices of a saddle structure, when the periodic boundary conditions are considered through constraint equations for the corresponding opposite nodes.

The paper presents numerical simulation results demonstrating interesting effects such as anisotropy class violation and the appearance of mechanical and electrical auxetic properties, both full and partial. These effects are caused by the properties of the initial dense piezoelectric materials, cell geometry, and inhomogeneous polarization. The report draws on recent results of investigations of piezoelectric metamaterials $(10.1016/j.actamat.2025.120744, 10.1007/s00707-$

[1]This research was supported by the Russian Science Foundation, grant number 22-11-00302-P, https://rscf.ru/project/22-11-00302/ at the Southern Federal University.

$025-04352-3, 10.1007/978-3-032-11462-4_9, 10.1007/s11029-026-10354-6)$, as well as on new studies.

HIGH-ORDER FINITE-DIFFERENCE SCHEMES AND THE INHERITANCE OF CONVECTIVE PROBLEM PROPERTIES[1]

Vyacheslav G. Tsybulin, Alexander A. Selischev
I.I. Vorovich Institute of Mathematics, Mechanics and Computer Science, Southern Federal University
8/a Milchkova str., Rostov-on-Don 344090, Russia
vgcibulin@sfedu.ru, selishev@sfedu.ru

Abstract Preserving structural properties of differential equations is essential for reliable numerics. We discuss applications of the finite-difference method to variable coefficient problems in wave propagation and convection in a porous medium. We derive compact finite-difference schemes that increase the order of approximation without widening the stencil. First, we analyze the 1D wave equation, deriving a three-point compact scheme via the balance method, and investigate several approaches to the associated spectral problem. Then we construct a nine-point compact scheme for planar Darcy convection that preserves cosymmetry for constant porosity. In Darcy convection, a nontrivial continuous family of stationary solutions arises because of the problem's cosymmetry. Computational experiments for permeability varying with the vertical coordinate demonstrate fourth-order convergence and correct reproduction of solutions, confirming the scheme's efficiency and accuracy.

[1]The research was funded by a Southern Federal University

NUMERICAL MODELING OF TEMPERATURE FIELDS FOR PREDICTING ACTIVE LAYER THICKNESS EVOLUTION UNDER PILE FOUNDATIONS IN PERMAFROST REGIONS WITH SEASONAL THERMAL STABILIZATION UNDER CLIMATE WARMING[1]

Natalya A. Vaganova

Krasovskii Institute of Mathematics and Mechanics, Ekaterinburg, Russia

16 S.Kovalevskaya Str., Yekaterinburg, 620077 Russia

fmy@imm.uran.ru

The bearing capacity of soils under capital facilities on pile foundations in permafrost regions is governed by the ground thermal regime. The evolution of active layer thickness (ALT) serves as a key indicator of changes in pile bearing capacity, since the major contribution to bearing capacity comes from the pile segments embedded in soil with negative temperatures. To reduce ALT beneath building foundations, various cooling devices are employed. The most widespread for ground thermal stabilization under residential buildings are seasonal cooling devices (SCDs), whose efficiency depends on the air temperature in ventilated basements as well as on the technical condition of the SCDs themselves. In the context of global climate warming, a long-term forecast of ALT evolution – and thus of the dynamics of pile bearing capacity – becomes critical. To address this, numerical modeling of ground temperature fields is used under different climate warming scenarios. The simulation results show that operating SCDs can maintain baseline ALT under moderate warming. However, under accelerated warming, ALT increases significantly regardless of SCDs operation, revealing threshold values of climate change beyond which pile bearing capacity falls below critical limits.

The study was conducted in collaboration with N.A. Vaganova, I.M. Filimonova, D. Zh. Shamugia, V.M. Kapkaev.

[1]The research was funded by a grant of the Russian Science Foundation N 25-21-00776.

PREDICTION OF AORTIC THROMBOSIS RISK BASED ON A COMPUTATIONAL MODEL: PLATELETS AND VON WILLEBRAND FACTOR[1]

Pavel A. Zelenchuk[1,2]

[1] *Scientific Center of Genetics and Life Sciences, Sirius University of Science and Technology,*
Sirius, 354349, Olympic Avenue 1, Russia.
[2] *Institute of Mathematics, Mechanics & Computer Science, Southern Federal University,*
Rostov on Don, 344090, Milchakova 8a, Russia.
E-mail: zelenchuk@sfedu.ru

Transcatheter aortic valve implantation is a minimally invasive surgical procedure that involves replacing a damaged aortic valve with an artificial one directly over the old one. After implantation of the prosthesis, zones of abnormally high shear stress develop in the aorta, which can cause stretching of von Willebrand factor (vWF) multimers, promoting their activation and platelet aggregation. vWF is a large multimeric glycoprotein in blood plasma that is essential for thrombus formation in arterial blood flow. Experimental studies of vWF dynamics are complicated by the microscopic time scales and flow rates involved. This research proposes a simple mathematical model that accounts for the concentration and degree of polymerization of vWF in the development of thrombosis.

The model demonstrates the existence of two stable stationary states: "health" (low level of active platelets, indicating no clot) and "thrombosis" (high level of active platelets, indicating a growing aggregate). The transition between these states occurs abruptly once the critical concentration of active von Willebrand factor is exceeded. The model is formulated as a low-dimensional system of ordinary differential equations that capture the key kinetic interactions between vWF multimers and platelets under pathologically elevated shear. Numerical analysis reveals that the threshold for the abrupt transition is strongly modulated by the local shear rate, suggesting that patient-specific flow simulations could refine individual risk assessment. These findings highlight the potential of monitoring vWF activity as an early computational predictor of thrombotic complications after transcatheter aortic valve implantation.

[1]The results were obtained with the financial support of the project "Development of technology for the design, numerical modeling and additive manufacturing of mesh structures for transcatheter implantation of the aortic valve" implemented within the framework of the state program of the federal territory "Sirius" "Scientific and technological development of the federal territory "Sirius" (Agreement No. 27-03 dated July 7, 2025).

ON A RELAXATION WEIGHTED GREEDY BLOCK KACZMARZ METHOD FOR SOLVING LINEAR SYSTEMS IN HILBERT SPACES

Zheng-Da Huang

School of Mathematical Sciences,
Zhejiang University,
No. 866 Yuhangtang Road,
Hangzhou 310058,
P.R. China
Email: zdhuang@zju.edu.cn

Abstract: In this talk, a relaxation weighted greedy block Kaczmarz (RWGBK) method is considered for solving the consistent system of bounded linear operators in Hilbert spaces. The method is deterministic and does not require computing the pseudoinverses of linear operators. It is proved under mild conditions that, for any initial point, the method will converge to a solution of the linear system nearest to the initial point, and that the iterative sequence starting from the zero point may converge linearly to the least norm solution. Numerical experiments on a application of the circular Radon transform are given to illustrate that the considered method is efficient and that it is necessary to consider the relaxation parameter and weight.

AN ACCELERATED SURROGATE HYPERPLANE KACZMARZ METHOD BASED ON TWO-DIMENSIONAL SEARCH

Rui LI

Department of Mathematical Research,
College of Data Science,
Jiaxing University,
No. 118, Jiahang Road, Nanhu District,
Jiaxing 314001, Zhejiang Province,
P.R. China
Email: zuqiuziye@163.com

Abstract: An accelerated surrogate hyperplane Kaczmarz method based on two-dimensional search is proposed, which generates a new hyperplane by searching for the optimal weighted vector in a two-dimensional subspace. Theoretical analysis provides the convergence rate of the new method. Numerical experiments demonstrate that the new proposed Kaczmarz method is convergent and outperforms the original method in terms of both the number of iterations and computational time.

A BLOCK-SHIFTED CYCLIC REDUCTION ALGORITHM FOR SOLVING A CLASS OF QUADRATIC MATRIX EQUATIONS

Xu LI
Department of Applied Mathematics,
Lanzhou University of Technology,
Lanzhou 730050, P.R. China
Email: lixu@lut.edu.cn

Abstract: The cyclic reduction (CR) algorithm is an efficient method for solving quadratic matrix equations that arise in quasi-birthвЂ“death (QBD) stochastic processes. However, its convergence is not guaranteed when the associated matrix polynomial has more than one eigenvalue on the unit circle. To address this limitation, we introduce a novel iteration method, referred to as the Block-Shifted CR algorithm, which improves the CR algorithm by utilizing singular value decomposition (SVD) and block shift-and-deflate techniques. This new approach extends the applicability of existing solvers to a broader class of quadratic matrix equations. Numerical experiments demonstrate the effectiveness and robustness of the proposed method.

(This is a joint work with Beatrice Meini)

ON COLUMN-BLOCK ENHANCED RANDOMIZED EXTENDED KACZMARZ METHOD FOR SOLVING INCONSISTENT LINEAR SYSTEMS

Cun-Qiang Miao
School of Mathematics and Statistics,
Central South University,
No. 932, Lushan South Road
Changsha 410083, P.R. China
Email: cqmiao@csu.edu.cn

Abstract: The randomized extended Kaczmarz method (SIAM J. Matrix Anal. Appl., 34: 773-793, 2013) proposed by Zouzias and Freris is highly effective for solving large-scale inconsistent linear systems. However, when the coefficient matrix is column-rank deficient or contains many linearly dependent columns, the method tends to select such linearly dependent columns, slowing down the convergence rate. In this talk, we derive an efficient block iteration scheme for column iteration steps of the randomized extended Kaczmarz method and construct a column-block enhanced randomized extended Kaczmarz method. We further establish the convergence theory of the column-block enhanced randomized extended Kaczmarz method, which exhibits faster convergence rate than the original randomized extended Kaczmarz method. Finally, numerical experiments are conducted to validate the effectiveness of our approach in terms of both iteration counts and computing times.

A MULTI-STEP GREEDY AUGMENTED KACZMARZ METHOD FOR SOLVING LARGE SPARSE INCONSISTENT LINEAR SYSTEMS

Ai-Li Yang

School of Mathematics and Statistics,
Hainan Normal University,
No. 99, Longkun South Road,
Haikou 570000,
P.R. China
Email: yangaili@hainnu.edu.cn

Abstract: To efficiently solve large-scale inconsistent sparse systems of linear equations, we first equivalently transform them into a class of block two-by-two consistent linear systems. Then, by effectively leveraging the special block structure of the coefficient matrix in these consistent linear systems, we propose a multi-step greedy augmented Kaczmarz (MGAK) method. This method devises a simple yet effective selection rule for working rows (i.e., hyperplanes), ensuring that the angles between working rows chosen in adjacent iterative steps are as large as possible, thereby enhancing the convergence rate of the method. The convergence properties of the method are analyzed in detail, and numerical experimental results validate its robustness and effectiveness.

A SPLITTING-BASED KPIK METHOD FOR EDDY CURRENT OPTIMAL CONTROL PROBLEMS

Min-Li Zeng

School of Mathematics and Finance, Putian University

Abstract: In this talk, we will explore efficient methods for discretized linear systems that arise from eddy current optimal control problems utilizing an all-at-once approach. We propose a novel low-rank matrix equation method based on a special splitting of the coefficient matrix and the Krylov-plus-inverted-Krylov (KPIK) algorithm. First, we reformulate the resulting discretized linear system into a matrix equation format. Then, by employing the KPIK algorithm, we derive a low-rank approximation solution. This new approach is referred to as the splitting-based Krylov-plus-inverted-Krylov (SKPIK) method. The SKPIK method exhibits the potential for efficiently tackle large and sparse discretized systems, while also significantly reducing both storage requirements and computational time. Next, theoretical results regarding the existence of low-rank solutions are provided. Furthermore, numerical experiments are conducted to demonstrate the effectiveness of the proposed low-rank matrix equation method in comparison to several established classical efficient techniques.

ACCELERATION OF THE MODULUS-BASED ITERATION METHODS BY GEOMETRICALLY SMOOTHED MOMENTUM FOR SOLVING LINEAR COMPLEMENTARITY PROBLEMS

Jian-Jun Zhang

Department of Mathematics,
College of Sciences,
Shanghai University,
No. 99, Shangda Road,
Shanghai 200444,
P.R. China
Email: zhangjj@shu.edu.cn

Abstract: In this paper, we propose to use geometrically smoothed momentum to accelerate the convergence rate of the modulus-based matrix splitting iteration methods for solving linear complementarity problem. Convergence of the proposed methods is proved when the system matrix is positive-definite or H_+ matrix. We compare the performance of the modulus-based methods with and without acceleration by geometrically smoothed momentum on a practical problem. The results demonstrate that modulus-based iteration methods with geometrically smoothed momentum acceleration converge much rapidly.

Invited talks

A TEMPORAL PROXIMITY REGULARIZED CLUSTERWISE VECTOR AUTOREGRESSIVE METHOD FOR MULTIVARIATE TIME SERIES CLUSTERING

Min LI

School of Mathematics and Physics,
Lanzhou Jiaotong University,
No.88, Anning West Road, Anning District,
Lanzhou 730070, P.R. China
Email: lim19@mail.lzjtu.cn

Abstract: Repeated patterns often appear in time series data. Discovering these patterns is challenging because time series need to be segmented and clustered simultaneously. Clusterwise regression is a useful approach that enables the segmentation and clustering of the time series simultaneously. In this paper, we consider the multivariate time series generated by vector autoregressive (VAR) models. By introducing a weight function matrix and using the clusterwise VAR method, we propose a minimization model that consists of the sum of weighted squared regression errors and the total variation of the weight function matrix. The total variation of the weight function matrix enforces temporal proximity on the clustering results. Based on alternating minimization, we design an expectation maximization (EM)-like method to estimate the weights and the VAR regression parameters alternatively. Experimental results for both synthetic and real-world time series show the effectiveness of the proposed method and its competitiveness compared with existing methods.

ON CONVERGENCE RATE OF THE RANDOMIZED MULTIPLICATIVE SCHWARZ METHOD

Lu WANG
School of Mathematical Sciences,
Hebei Normal University,
Shijiazhuang 050024, P.R. China
Email: wanglu@hebtu.edu.cn

Abstract: The randomized coordinate descent (RCD) method and the randomized Kaczmarz (RK) method are two classical and widely-used randomized iteration methods for solving large-scale linear systems, they can be regarded as two special cases of the randomized multiplicative Schwarz (RMS) method. Motivated by the exact error analysis of the RCD method and the RK method, we conduct a closed-form formula for the solution error of the RMS method and further give a tighter upper bound for its convergence rate. On this basis, we unify the convergence results of the RCD method and the RK method, and generalize them to the more general extrapolated RCD method and extrapolated RK method. Numerical experiments confirm that the new estimate for the convergence rate of the RMS method is more accurate, and it can help us to find a more proper relaxation parameter.

TWO NEW BLOCK PRECONDITIONERS FOR DOUBLE SADDLE POINT PROBLEMS ARISING FROM LIQUID CRYSTAL DIRECTOR MODELING

Bo WU

Ningxia University

Abstract: In this talk, we design two iterative methods based on matrix splitting techniques to solve double saddle point problems arising from the liquid crystal director model. These iterative methods are shown to be unconditionally convergent under certain conditions on the Schur complement S. We further propose new block diagonal (NBD) and new block triangular (NBT) preconditioners derived from these iterative schemes, and analyze the spectral properties of the corresponding preconditioned matrices. Finally, numerical experiments are carried out to validate the robustness and efficiency of the proposed iterative methods and preconditioners for solving double saddle point problems in liquid crystal director modeling.

A MODIFIED PRECONDITIONED QUASI SHIFT-SPLITTING METHOD FOR DOUBLE SADDLE POINT PROBLEMS WITH LIQUID CRYSTAL DIRECTOR MODELING

Wei-Hong Zhang

Department of Mathematics,
School of Mathematics and Physics,
Lanzhou Jiaotong University,
No.88, Anning West Road,
Lanzhou 730070, P.R. China
Email: zwhmath09@163.com

Abstract: For the double saddle point problems (DSPPs) that arise in liquid crystal director modeling, we introduce a modified preconditioned quasi shift-splitting (MPQSS) iteration method. The convergent properties of the MPQSS iteration algorithm are carefully analyzed. Based on the MPQSS iterationmethod, two quasi shift-splitting (SS) preconditioners are developed. The spectral properties of the preconditioned DSPP matrices with these new preconditioners are thoroughly investigated. The implementations of the new preconditioners avoid direct and precise construction of the relevant Schur complement matrices, thereby enhancing the efficiency of the algorithms. Mathematical tests validate the reliability and effectiveness of the new methods by assessing the performance of the Krylov subspace method after being preconditioned with the new preconditioners.

(Joint work with Sheng-Wei Zhou and Mei-Ling Zhang)

DISCRETE EVENT SIMULATION FOR LMS-DRIVEN EXTENDED REALITY MODELING OF PRODUCTION PROCESSES

Vadim A. Afanasyev, Sergey A. Tyasto
Moscow State University of Technology «STANKIN»,
Moscow, Russia
Email: dfjx2201@yandex.ru

Nowadays technological progress is moving forward at a rapid pace and so do production process complexity. As workloads grow it becomes difficult to prepare both equipment and qualified staff. Simulation modeling methods come in handy for cases like this providing an opportunity to enhance the readiness of manufacturing workforce and capacity. In this study we consider the use of discrete event simulation (DES) in the context of extended reality applications based on the learning management system (LMS) approach. [1]

A vital part of the Republic of Sakha (Yakutia) economy is mining. Diamond production is one of major industries in this region which implies active development of mines and corresponding machinery. Such processes as diamond-bearing ore transportation, primary processing and dressing are distinguished in this field therefore DES models can be integrated as core predictive technology capable of optimization before evaluation. [2]

For instance, the system state may reflect the flow of materials, mining and processing equipment status, the state of infrastructure. The processes mentioned earlier can be described using stochastic DES models of queueing type. At this point it is also important to monitor staff professional development. LMS data is supposed to be recorded in extended reality (XR) sessions. VR/AR-approaches provide multiple advantages such as visualizing data in real time, leveraging the sense of presence and deliver immersive experience without risks including health harm and equipment rearrangement costs. The system discussed is developed using game framework Unreal Engine 5 featuring realistic 3D-graphics and wide variety of tools designed for different subsystem implementation. [3]

Bibliography

1. *Liang Gong* Virtual Reality Technology for Factory Layout Planning // Chalmers University of Technology. – 2018.

2. *Giulia Wally Scurati, Francesco Ferrise, Marco Bertoni* Extended realities and discrete events simulations: A systematic review to define design trade-offs and directions // Computes in Industry. – 2025. – Vol. 164.

3. *Yiran Yang, Shuchisnigdha Deb, Miao He, Md Humaun Kobir* The use of virtual reality in manufacturing education: State-of-the-art and future directions // Manufacturing Letters. – 2023. – Vol. 35.

INCREASING THE ORDER OF APPROXIMATION OF DIFFERENCE SCHEMES BY CORRECTING THE RIGHT-HAND SIDE USING THE ORIGINAL SOLUTIONS[1]

Roman A. Golubev, Vladimir V. Shaydurov, Lidiya V. Gileva

Institute of Computational Modeling of Siberian Branch of Russian Academy of Science;

Russia, Krasnoyarsk

Email: rgolubev@icm.krasn.ru

Abstract

The paper examines an approach to improving the accuracy of difference schemes for one- and two-dimensional parabolic equations. The order of approximation is increased by adjusting the right-hand side of the difference scheme without changing its first differential approximation or the grid operator. The adjustment is performed by adding a term to the right-hand side of the difference scheme that is the time derivative of the original solution of a lower order of accuracy [1]. Theoretical foundation and computational experiments is conducted for the resulting modified difference schemes.

Bibliography

1. *Bohmer K., Stetter H.J.* Defect correction methods. Theory and Applications, Springer:Wien; 1984: 243.

[1]This work is supported by the Krasnoyarsk Mathematical Center and financed by the Ministry of Science and Higher Education of the Russian Federation in the framework of the establishment and development of regional Centers for Mathematics Research and Education (Agreement No. 075-02-2026-735).

NUMERICAL POLYGONAL MASK GENERATION WITH CONTROLLABLE GEOMETRY AND TOPOLOGY FOR BENCHMARKING INPAINTING ALGORITHMS

Mikhail M. Karyakin, Yana M. Demyanenko

Southern Federal University,

Vorovich I. I. Institute of Mathematics, Mechanics and Computer Science,

Rostov-on-Don, Russia

Email: mkariakin@sfedu. ru, demyana@sfedu. ru

The task of restoring image structural integrity (inpainting) is limited by the lack of representative datasets with complex mask geometries, since most existing datasets contain either primitive shapes (such as rectangles, ellipses, or strokes) or entirely random complex shapes. To comprehensively compare modern deep learning-based approaches (e.g., EdgeConnect, LaMa, and MAT), a mask generator with user-specified parameters is essential.

We propose an algorithm for polygonal mask generation, controlled by three parameters:

1. Aspect ratio α: the ratio of the shorter to the longer dimension of the bounding box ($\alpha = 1$ for a square)
2. Raggedness r: $r = P2/(4\pi S)$, with P and S being the perimeter and area, respectively, and the factor 4π scales the metric to the interval $[1, \infty)$
3. Genus g: the number of holes in the mask

Our numerical approach consists of the following steps:

1. Generation of a base convex polygon B inside a bounding box with the specified aspect ratio α
2. Iterative approximation of the specified raggedness
   (a) Deforming the border via subtraction of random small convex polygons along the perimeter, thereby increasing its length and reducing the area
   (b) Generating the specified number of holes: for each hole, a deformed polygon h is generated, followed by the same subtraction operation $B := B/h$

To achieve a more precise raggedness approximation, we employ analytical hole scaling, where the target scaling factor is derived from a quadratic equation relating it to the target raggedness and the hole's own raggedness, as well as to the current perimeter and area of the mask.

Each generated mask was applied to three images from the Places2 dataset; then, the inpainting quality of the EdgeConnect architecture was evaluated using PSNR, SSIM, and LPIPS metrics. Correlation analysis revealed a strong inverse relationship between the model's performance and both mask size and aspect ratio α, whereas a direct correlation was observed with mask raggedness. The impact of the topological genus on this specific implementation is weak due to the generator's constraints at high raggedness levels.

DYNAMICS AND DATA: PARAMETER FITTING IN THE LIF NEURON MODEL

Xiaolong Liu, Galina V. Muratova
Vorovich Institute of Mathematics, Mechanics and Computer Science, Southern Federal University
8a Milchakova St., Rostov-on-Don 344090, Russia
tliu@sfedu.ru; muratova@sfedu.ru

Abstract Computational neuroscience aims to understand brain information processing through mathematical models. A central challenge is balancing biological realism with computational efficiency. The Hodgkin-Huxley (HH) model is accurate but too complex for large networks. Hence, simpler models like the leaky integrate-and-fire (LIF) model are widely used.

The LIF model describes sub-threshold membrane dynamics as a linear RC circuit:

$$C_m \frac{dV}{dt} = -g_L(V - V_{rest}) + I(t)$$

C_m denotes the membrane capacitance, g_L the leak conductance, V_{rest} the resting potential, and I(t) the injected current. For constant current, it has an analytical solution, making parameter estimation straightforward.

A model's usefulness isn't just about its theoretical structure – it also hinges on whether it can be effectively validated and calibrated against experimental data. This brings us to the problem of data-driven parameter fitting. Key parameters of the LIF model, such as the resting potential V_{rest} and the membrane time constant $t_m = C_m/g_L$, need to align with the biophysical properties of real neurons.

We use mean squared error (MSE) as a loss function:

$$L(V_{rest}, T_m) = \frac{1}{N} \sum_{(} i = 1)^N (V^{data}(t_i) - V^{model}(t_i; V_{rest}, T_m))^2$$

Optimization yields the best-fit parameters:

$$(V^*_{rest}, T^*_m) = arg \min_{V_{rest}, T_m} L(V_{rest}, T_m)$$

However, the LIF model has inherent limitations. Its linear passive dynamics cannot capture complex behaviors like subthreshold oscillations, bistability, or spike-frequency adaptation. Forcing such data into LIF fitting may still converge, but often produces biologically meaningless parameters. This is not a fitting failure but a sign of model inadequacy.

Thus, data-driven parameter fitting serves as a critical bridge between abstract models and real neurons. It reveals not only when a model works, but more importantly when it breaks down – guiding the need for richer models (HH, FitzHugh-Nagumo). This interplay between dynamics and data lies at the core of modern computational neuroscience.

EXPERIMENTAL STUDY OF BLOCK SIZE AND CUTHILL-MCKEE-TYPE REORDERING EFFECTS ON GPU BLOCK-SPARSE LINEAR SOLVERS

Pavel A. Oganesyan1[2], Ivan D. Ignatenko[2,*]., Vladislav V. Krylov[2], Yaqi Liu[1]

[1] *Shenzhen MSU-BIT University, Shenzehn, China*

[2] *Southern Federal University, Rostov-on-Don, Russia*

[*] *Presenting Author*

email: oganesyan@hey.com, iignatenko@sfedu.ru

This work presents an experimental study of how the block structure of sparse matrices affects the convergence and practical efficiency of iterative linear solvers in a GPU-oriented block-sparse framework. The study is based on the numerical package developed by the authors, BSMP (Block-Sparse Matrix Package), a CUDA/C++ implementation for block-sparse matrix storage and operations. The package provides the core infrastructure required for such experiments, including matrix loading, block-sparse matrix-vector products, and GPU-enabled Krylov solvers for linear systems. In particular, the current solver pipeline includes BiCGStab (BiConjugate Gradient Stabilized), restarted GMRES (Generalized Minimal Residual), and CG (Conjugate Gradient), together with several preconditioning options available in a unified interface: AMG (Algebraic Multigrid), scalar Jacobi, block Jacobi, and the unpreconditioned mode.

The package is primarily intended for problems with sparse matrices arising in mechanical and electromechanical applications. The structure of such matrices is often determined by the numbering of nodes in the underlying finite element mesh, which naturally induces a block structure in the resulting sparse system. In general, these matrices are highly sparse, with only a small number of nonzero entries per row, but their bandwidth can still be large when no dedicated reordering is applied. In our previous work, we considered geometry-level reordering strategies aimed at reducing bandwidth and improving the convergence of iterative solvers. In the present study, we implement a block-sparse bandwidth reduction strategy based on the Reverse Cuthill-McKee (RCM) algorithm, a classical and widely used heuristic. We then compare the behavior of the implemented solvers on the original and reordered matrices using well-known benchmark problems, specially generated test problems from static mechanical and electromechanical models, and a large-scale real-world problem from the mechanical domain.

The obtained results indicate that matrix block structure has a substantial influence on solver convergence, and that RCM-based reordering can lead to noticeable convergence improvements for a number of problems.

SINGLE-KERNEL PREFIX SUM WITH GRID SYNCHRONIZATION FOR K-D TREE CONSTRUCTION ON GPU

Maxim A. Vavilkin, Yana M. Demyanenko

Southern Federal University,

Vorovich I. I. Institute of Mathematics, Mechanics and Computer Science,

Rostov-on-Don, Russia

Email: vavilkin@sfedu.ru, demyana@sfedu.ru

In mathematical modeling tasks, such as photon mapping for caustic simulation, efficient processing of large multidimensional datasets is essential. To achieve this, spatial acceleration structures are commonly used. One of the most popular options is the k-d tree. For constructing such trees on the GPU, a parallel radix sort applied to elements at each tree level is an effective approach. A radix sort algorithm, in turn, relies on a prefix scan (prefix sum) operation. However, conventional GPU implementations of a prefix sum (GPU Gems 3) require multi-pass merging for large arrays, which leads to multiple kernel launches and increased overhead.

The purpose of this work is to develop a single-pass parallel prefix sum implementation on CUDA that eliminates sequential kernel launches for arrays of arbitrary length, and to apply it to the left-balanced k-d tree construction with a minimal amount of temporary memory.

This is achieved by:

- Using a grid-wise synchronization mechanism instead of recursive or iterative merging of partial scan results.
- Leveraging C++ templates (element type, inclusive/exclusive sum, a flag for storing the total sum in a dedicated variable) and a unified access interface to shared memory of arbitrary types through a custom SharedMem<T> structure.
- Enabling the algorithm to process arrays of arbitrary length in a single kernel launch without additional merging steps, by internally calling the standard limited-size prefix sum functions proposed in GPU Gems 3.

The modified prefix sum algorithm is then applied to radix sort, which is used for the level-by-level construction of the k-d tree. Specifically, it is used to sort 3D points along axes that are automatically selected at each tree level. Since the sort operates on 32-bit floating-point coordinates, the radix sort invokes the modified prefix sum 32 times per pass (once per bit). This approach allows:

- Processing elements of each subtree without control-flow divergence.
- Computing offsets for already processed elements and moving them to the beginning of the input array with a single prefix sum call per tree level.
- Achieving an overall $O(nlog_2 n)$ k-d tree construction complexity with two kernel launches per tree level.

As a result, the tree construction algorithm requires additional temporary memory of $n15bytes$. For comparison, an input array of $3D$ floating-point coordinates occupies $n12bytes$, while the full photon structure that is used in testing (which includes these points) takes 36 bytes per element. The algorithm is easily extensible to arbitrary dimensions and data types. The resulting heap-indexed left-balanced k-d tree is suitable for accelerating nearest-neighbor searches on both GPU and CPU. Furthermore, the proposed prefix sum method can be applied in other algorithms requiring massive parallel scans, such as histograms, stream compaction, and scientific computations.

Thus, we present a prefix sum implementation with grid-wise synchronization that overcomes the limitations of the traditional multi-pass approach. Based on this, a memory-efficient left-balanced k-d tree is constructed requiring only two kernel launches per level. The resulting pipeline is well-suited for integration into numerical modeling and parallel computing systems.

ANALYTICAL AND NUMERICAL INVESTIGATION OF TURING INSTABILITY IN A REACTION-DIFFUSION PREDATOR-PREY MODEL[1]

Pavel A. Zelenchuk[1,2]

[1] *Scientific Center of Genetics and Life Sciences, Sirius University of Science and Technology,*
Sirius, 354349, Olympic Avenue 1, Russia.
[2] *Institute of Mathematics, Mechanics & Computer Science,*
Southern Federal University,
Rostov on Don, 344090, Milchakova 8a, Russia.
E-mail: zelenchuk@sfedu.ru

The emergence of spatially heterogeneous structures (Turing patterns) in distributed biological systems remains a fundamental problem in mathematical biology. Of particular interest are the conditions for diffusion-driven instability, whereby a system that is stable in the absence of diffusion becomes unstable when spatial transport is introduced. Such mechanisms underlie the explanation of morphogenesis and non-uniform population distributions. The conditions for the onset of classical Turing (diffusion-driven) instability have been investigated analytically for the system

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 \frac{\partial^2 u}{\partial x^2} + u[\mu_1 (u-a)(1-\frac{u}{p}) - \lambda_1 \frac{v}{p}], \\ \frac{\partial v}{\partial t} = d_2 \frac{\partial^2 v}{\partial x^2} + v[\mu_2 (1-\frac{u}{p}) - \lambda_2 - \gamma v] \end{cases} \quad (1)$$

For the case of a homogeneous resource, constraints on the system parameters were derived under which the coexistence steady state is stable in the absence of diffusion but loses stability when spatial transport with substantially different diffusion coefficients is introduced. It is shown that the key requirements are $f_u < 0$ and $d_2 \gg d_1$, where f_u is the partial derivative of the prey kinetics evaluated at equilibrium, and d_1, d_2 are the diffusion coefficients of the prey and predator, respectively. When these conditions are violated, the classical Turing mechanism does not operate, and any spatial patterns observed must stem from a different origin. Additionally, the effect of a spatially heterogeneous resource was examined: in the regime where the characteristic scale of the heterogeneity greatly exceeds the Turing wavelength, the resulting patterns become modulated by the resource background. This prediction is corroborated by numerical experiments.

[1] This research was supported by the Russian Science Foundation, grant number 25-21-00419 at Sirius University of Science and Technology, https://rscf.ru/project/25-21-00419/

ON CONSTRUCTION OF A COMPACT FOURTH-ORDER SCHEME FOR THE GENERAL ACOUSTIC WAVE EQUATION[1]

Alexander A.Zlotnik, Timofey A. Lomonosov

Higher School of Economics University, Moscow, Russia

We deal with the general multidimensional acoustic wave equation, where both the speed of sound and density of a medium are variable, that is physically relevant. For the first time, we develop a new compact scheme for this equation which is three-level in time, three-point in each spatial direction and has the fourth truncation order. In addition, the scheme is semi-explicit in time and easily implementable. It non-trivially generalizes a semi-explicit vector compact scheme constructed and studied recently in the much simpler case of the constant density. Although other high-order numerical methods for the general acoustic wave equation were known, the construction of a compact high-order scheme remained an unsolved problem for a long time.

Results of computations confirm the high precision of the scheme and its fourth error order not only in the mesh C norm but in the mesh C^1 norm as well that is of value in applications.

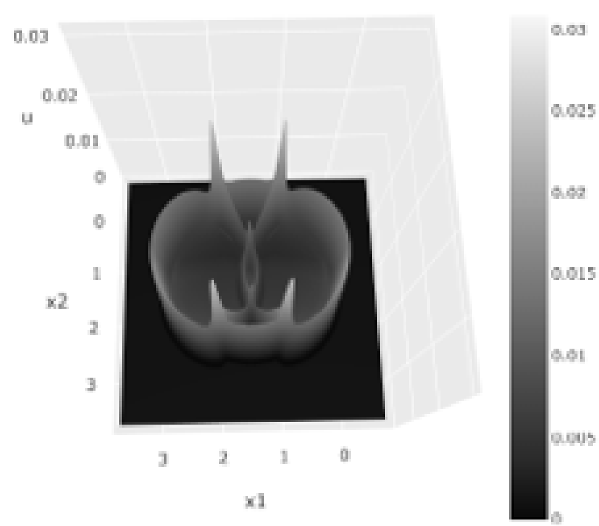

Figure 1. The 3D graph of the wavefield at $t = 1.15$

[1]The study was implemented in the framework of the Basic Research Program at HSE University in 2026 year.

Bibliography

1. *L.M. Brekhovskikh and O.A. Godin* Acoustics of Layered Media I. Plane and Quasi-Plane Waves, Springer, Berlin, 2012.

2. *J. Dubois, S. Imperiale, A. Mangeney, F. Bouchut, and J. Sainte-Marie* Acoustic and gravity waves in the ocean: a new derivation of a linear model from the compressible Euler equation. J. Fluid Mech., 970, article 28 (2023)

3. *A. Zlotnik, T. Lomonosov* On stability and error bounds of an explicit in time higher-order vector compact scheme for the multidimensional wave and acoustic wave equations. Appl. Numer. Math., 195, 54-74 (2024)

Scientific publication

NUMERICAL ALGEBRA WITH APPLICATIONS

Proceedings of Lecturers and Young Scientists

5-11 July 2026
Krasnoyarsk – Rostov-on-Don, RUSSIA

Подписано в печать 26.06.2026 г.
Бумага офсетная. Формат 60×84/16. Тираж 50 экз.
Усл. печ. лист. 2,79. Уч.-изд. лист. 2,0 Заказ № 10496.

Отпечатано в отделе полиграфической, корпоративной
и сувенирной продукции Издательско-полиграфического комплекса ЮФУ.
344090, г. Ростов-на-Дону, пр. Стачки, 200/1, тел. (863) 243-41-66.